Liceo Scientifico Statale M.Grigoletti, Pordenone
Modulo CLIL sui Numeri Complessi
Homework - Lesson 9

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Finding the square root of a complex number without using the polar form: an example to study.

Find the square roots of $1 + i$. If a complex number, $z = x + iy$ is the square root of $1 + i$, one must have: $(x + iy)^2 = 1 + i$, that is $x^2 + 2ixy - y^2 = 1 + i$. This is equivalent to the solution of the system:

$$\begin{cases} x^2 - y^2 = 1 \\ 2xy = 1 \end{cases}$$

The solution is straightforward (find $x = \frac{1}{2y}$ from the second equation and substitute it in the first one). You get:

$$\begin{cases} x = \pm\sqrt{\frac{1+\sqrt{2}}{2}} \\ y = \pm\frac{1}{\sqrt{2+2\sqrt{2}}} \end{cases}$$

And now the exercises. Factorize the following polynomials.

1. $z^4 - i$

2. $z^5 + 243$

3. $z^3 + 64i$

4. $z^3 + 1$

5. $z^4 - 81$

6. $z^6 - 64i$

7. $z^4 + (1 + i)$

8. $z^3 - (1 - i)$

9. $z^2 - 2z + i$

10. $z^2 - iz + i$