Università Ca' Foscari di Venezia - Dipartimento di Economia - A.A.2016-2017

Mathematics (Curriculum Economics, Markets and Finance)

Tip: Reducing inequalities

Luciano Battaia

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One of the most important techniques used to solve inequalities is the possibility of simplifying both members by a common factor. The rule can be expressed as follows:

If both the members of an inequality have a common factor you can reduce the inequality by cancelling this common factor, without any other change, if and only if *this factor is strictly positive, that is* greater than but *not* equal to 0.

It is important to observe that this common factor must not necessarily be a number: it can also contain the variable, provided that it is strictly positive. For instance you can cancel x^2+1 as common factor, but you *cannot* cancel x^2: x^2+1 is greater than 0 (and never 0), while x^2 can also be 0. Let us consider two examples.

Example 1. *Given the inequality*

$$(1+x^2)(x-1) \leq (1+x^2)(2-3x),$$

you can proceed as follows.

$$\cancel{(1+x^2)}(x-1) \leq \cancel{(1+x^2)}(2-3x) \quad \Rightarrow \quad x-1 \leq 2-3x \quad \Rightarrow \quad x \leq \frac{3}{2}.$$

Example 2. *The two inequalities*

$$x^2(x-1) \geq 0 \quad \textit{and} \quad x-1 \geq 0$$

are not *equivalent: the first has*

$$\{0\} \cup [1, +\infty[$$

as the set of solutions, while the second has only

$$[1, +\infty[$$

as the set of solutions.