# Mathematics (Economics, Markets and Finance)

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## Exercises sheet 1

**Exercise 1.** *Find the natural domain of the functions*

$$f(x)=\sqrt{\frac{x^3}{x-1}},\quad g(x)=\sqrt[3]{\frac{x^3}{x-1}},$$

*and explain why the domain is different for the two functions.*

**Exercise 2.** *Find the natural domain of the function*

$$f(x)=\sqrt{x+1}+\sqrt{2-x}.$$

**Exercise 3.** *Find the natural domain of the function*

$$f(x)=\ln(x^2-3x+2).$$

NB*: "*ln*" means "natural logarithm".*

**Exercise 4.** *Plot the graph of the following function*

$$f(x)=\begin{cases} x^2, & \text{if } x<-1;\\ x+2, & \text{if } -1\le x\le 1;\\ -x+2, & \text{if } x>1. \end{cases}.$$

*Using the graph, find the limits of this function as $x$ tends to $-1$, $1$, $-\infty$, $+\infty$. Is this function one-to-one? Find the range of this function.*

**Exercise 5.** *Given the function*

$$f(x)=x^3-x^2-\frac{x^4}{4},$$

*a) find the natural domain;*
*b) find the subset of $\mathbb{R}$ where the $f(x)>0$;*
*c) find the limits*

$$\lim_{x\to-\infty} f(x),\quad \lim_{x\to+\infty} f(x);$$

*d) compute the first derivative and find where this derivative is positive or negative.*

**Exercise 6.** *Compute the first derivative of the following functions.*

*a)* $f(x) = 4x^2 - \frac{7}{x^2} + 5\sqrt[3]{x}$

*b)* $f(x) = x \ln x - x + 1$

*c)* $f(x) = e^x \ln x$

*d)* $f(x) = \frac{4x^4 - 7}{1 + x^2}$

*e)* $f(x) = \frac{\ln x + x}{x + \sqrt{x}}$

*f)* $f(x) = e^{\frac{x}{x+1}}$

*g)* $f(x) = \sqrt{1 + \ln x}$

*h)* $f(x) = (1 + x^2)^2$

*i)* $f(x) = (1 + x^2)^{12}$

*j)* $f(x) = x^2 \ln(xe^x)$

*k)* $f(x) = \frac{1}{x^4 + 1}$

*l)* $f(x) = x\sqrt{x^2 + 1}$

**Exercise 7.** *Say whether the following function is continuous*

$$f(x) = \begin{cases} -2x, & \text{if } x < 0; \\ 2x, & \text{if } 0 \leq x \leq 1; \\ 3, & \text{if } x > 1. \end{cases}$$

**Exercise 8.** *Say for what values of the real parameters a and b the following function is continuous and differentiable*

$$f(x) = \begin{cases} \ln x, & \text{if } x \geq 1; \\ ax - bx^2, & \text{if } x < 1. \end{cases}$$

**Exercise 9.** *Say for what values of the real parameters a and b the following function is continuous and differentiable*

$$f(x) = \begin{cases} ax, & \text{if } x \leq -1; \\ 2 - bx^2, & \text{if } x > -1. \end{cases}$$